Effective Low-Pass Filter Function Due to the Use of a Comb Filter and Virtual Oversampling

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***Abstract*—To allow for sophisticated signal processing algorithms in the receiver of a localization system that operates according to the multilateration principle, it is valuable to know the shape of signal transitions. However, if comb filters are used during digital signal processing and the effective sampling rate is increased by virtual oversampling, then frequency offsets of the modulation signal's harmonics will alter such transitions. In this paper it is shown, that this effect can be modeled by an effective low-pass filter function in continuous-time domain.**

I. Introduction

Imagine you could track objects with 1 mm spacial and 2 ms temporal resolution using small form factor transceivers attached to them. What would you do with such a capability? The analysis of human motion as discussed in [1] is one example that could take advantage of such technology. In athletic performance analysis and medical diagnostics - two applications mentioned in [1] - transceivers could replace optical markers that correspond to joints in stick figure representations of the human body. Since identification data can be provided along with 3D positions, the establishment of correspondence between model features and measurement data would be simplified. It is intuitive to extend the scope of applications to motion tracking in robotics or the movie industry. Other examples are man-machine interfaces, monitoring of a person's position in ambient assisted living as well as surveillance of small autonomous vehicles like drones.
A system that aims at achieving this precision was proposed in [2]. A possible, specific implementation is shown in Fig. 1. It operates according to the time difference of arrival (TDOA) multilateration principle and uses at least four and up to eight transmitters with fixed, known positions as stations. A mobile device uses a receiver with low intermediate frequency (low-IF) at which it branches into eight digital signal processing (DSP) channels.
All transmitters use identical bipolar clock signals to modulate their individual sinusoidal carriers. A peculiarity of the system is the channel spacing, which is much closer than the spacing of modulation signal harmonics. Fig. 2 shows the first nine harmonics of the bipolar clock signal around the individual carriers in the positive half of the spectrum for channels ① to ⑧ of which only odd harmonics are different from Zero. The signals generated in this way are emitted by the stations and ideally experience some propagation path delay (as well as attenuation) before they reach the receiver of the mobile device. The times of arrival (TOAs) can be extracted from the zero-crossings of the modulation signals, while the times of transmission (TOTs) remain unknown. Hence, TDOAs are used instead of TOAs to localize the receiver. In order to determine the TDOAs from the input signals, the receiver first applies some amplification (omitted in Fig. 1). Then, it converts the signals down to the intermediate frequencies by a complex down-conversion mixer. Due to this step, the spectrum in Fig. 2 centered around 5.8 GHz gets centered around DC. The negative half of the spectrum is shifted to -11.6 GHz and eliminated by the subsequent low-pass filter. Simultaneously, this filter acts as anti-aliasing filter for the analog-to-digital conversion that is performed next. In the digital domain, the signal is distributed to eight parallel DSP slices that perform the final down conversion by applying the CORDIC algorithm (This is symbolized by the second complex down conversion mixer in Fig. 1). Then channel separation is performed by three cascaded comb filters (lumped together in Fig. 1) each using an attenuation factor of $g = 0.99$. A cascade of three filters is required in this system to provide sufficient suppression of neighboring channels.
In order to determine the zero-crossings accurately, the principle of virtual oversampling is applied to increase the effective sampling rate. This is possible because of the periodicity of the modulation signal. A period length of $l_P = (nSpC \cdot nVOS - 1)/(nVOS \cdot f_S)$ is used for the signal in Fig. 2, where $nSpC$ is the average number of samples per clock period of the modulation signal, f_S is the sampling frequency, and $nVOS$ is the desired factor of sampling rate increase. Then, $nVOS$ periods of the modulation signal need to be collected coherently before one period of the modulation signal at increased sampling rate is available. For the implementation discussed here, $nSpC = 128$, $nVOS = 128$, and $f_S = 160\,\text{MHz}$. In this case, samples of successive periods are not in correct order to provide the signal at increased sampling rate directly but need to be permuted. This is the task of the final blocks in Fig. 1, which represent Perfect Shuffle permutations [3]. As the case may be, additional signal processing is utilized to improve the result of TDOA extraction.

II. Problem Statement and Analysis of Prior Art

Unfortunately, there is another consequence of virtual oversampling that is less obvious. A modified period length of the modulation signal is congruent with frequency offsets that shift

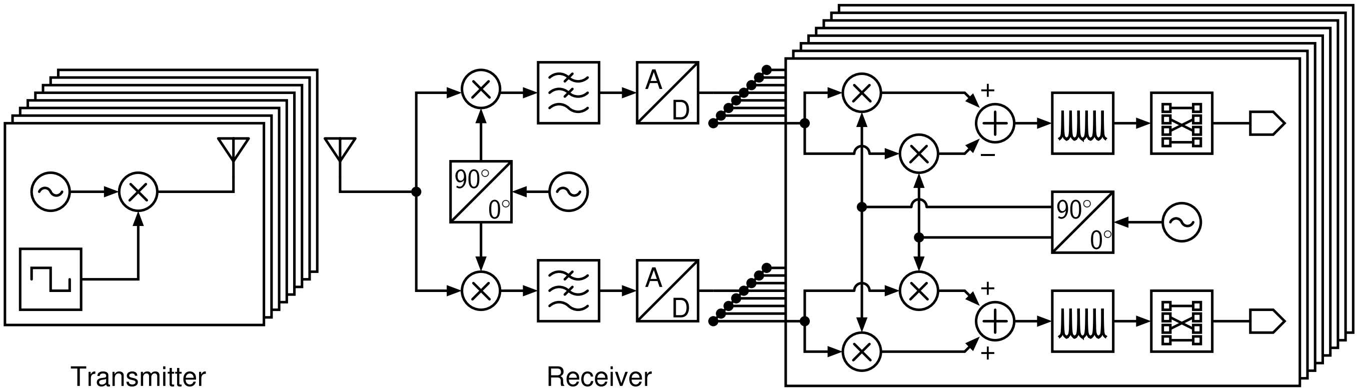

Fig. 1. Block level schematic of a specific implementation of a multilateration system as proposed in [2]. The system uses up to eight stations equipped with transmitters (left). Mobile devices use low-IF receivers that branch into eight DSP channels (right).

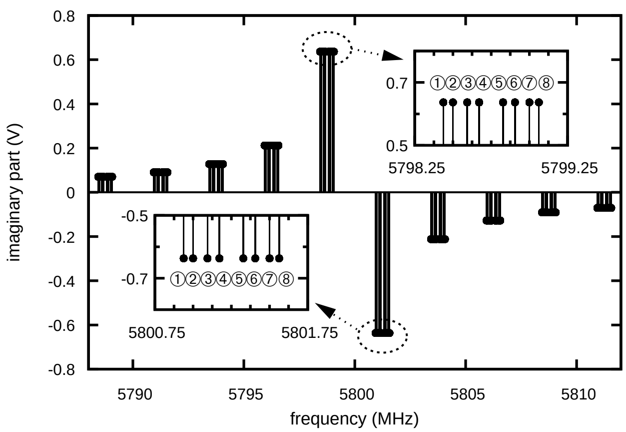

Fig. 2. First nine harmonics of the bipolar clock signal around the individual carriers of channels ① to ⑧ in the positive half of the spectrum. Only odd harmonics are different from Zero.

the harmonics of the modulation signal progressively out of the transmission band centers of the comb filters. The effect of this observation on the transitions of the modulation signal needs to be examined, as the assumption of ideal signal transmission does not apply. Especially indoors, transmission is affected by notable multi-path propagation. Some of the algorithms that try to suppress distortions caused by multi-path signals exploit the shape of these transitions, which requires a thorough characterization. In general, the effect of carrier offset on the comb filter's output signal is known from literature. Due to the in-band ripple of the orthogonal frequency-division multiplexing (OFDM) power spectral density as well as its shift caused by a carrier offset, [4] advocates the use of a comb filter for fine acquisition of carrier frequency. For this purpose, zeros of the comb filter are rotated on the unit circle in order to maximize its output power. The opposite objective, implying a suppression of contributions of active subscriber stations to the comb filter output power due to the zero placement, is formulated in [5] for frequency synchronization in the uplink of a orthogonal frequency-division multiple access (OFDMA) communication system with interleaved or generalized sub-carrier allocation. Similarly, it is highlighted in [6] that the GPS L1 signal possesses a "sinc-shaped" line spectrum convolved with the spectrum of slow data. Thus, use of a comb filter in conjunction with the conventional integrate-and-dump filter is proposed to accelerate the acquisition of weak signals and to improve the performance of the code tracking loop. In addition, a detailed discussion of comb filters is provided.

III. Effective Low-Pass Filter

In order to characterize the effect of the frequency offsets on the transitions of the modulation signal, first they need to be quantified. The offset f_{Off} of the modulation signal's fundamental frequency f_{Fund} from the center of the first transmission band of the comb filter f_{Center} is given by

$$f_{Off} = \underbrace{\frac{f_S \cdot nVOS}{nVOS \cdot nSpC - 1}}_{f_{Fund}} - \underbrace{\frac{f_S}{nSpC}}_{f_{Center}}. \quad (1)$$

This reduces to f_{Center} divided by $(nVOS \cdot nSpC - 1)$. However, for assessing the impact of the frequency offsets on the signal transitions, the ratio of f_{Off} to f_{Fund} is required rather than the ratio of f_{Off} to f_{Center}. It is given by

$$\frac{f_{Off}}{f_{Fund}} = \frac{1}{nVOS \cdot nSpC}. \quad (2)$$

Next we make use of the observation from [6] that a high pass filter can be turned into a comb filter by replacing a single delay by k delays. This is equivalent to a compression of the frequency axis by the same factor. In the present case, this step has to be reversed, so that we obtain a high pass filter from the comb filter. If we would apply the same stretch factor of $k = nSpC/2$ to the frequency spectrum of the modulation signal, the harmonics situated at $(2 \cdot m - 1) \cdot (f_{Center} + f_{Off})$ would alias to $-\mathrm{sgn}(2 \cdot m - 1) \cdot f_S/2 + k \cdot (2 \cdot m - 1) \cdot f_{Off}$. Still, their distance from the Nyquist frequency would be a factor $nVOS \cdot nSpC/k$ smaller than the actual distance of the harmonics from DC. Of course, this is only a thought experiment, i.e. the frequency spectrum will not be changed.

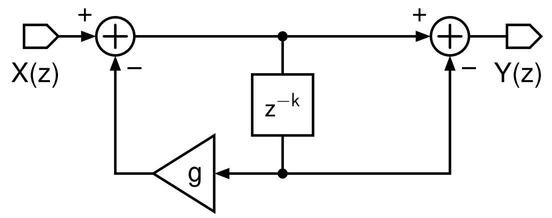

Fig. 3. Block level schematic of one of the comb filters used in Fig. 1.

However, for the high-pass filter gained by this substitution, this implies two things: It needs to be transformed into a low-pass filter and its bandwidth needs to be extended by a factor of $2 \cdot nVOS$. The first aspect is addressed in [7]. If the $3\,dB$ corner frequencies of the low pass, f_{lp}, and the high pass, f_{hp}, are related by

$$f_{lp} = \frac{f_S}{2} - f_{hp}, \tag{3}$$

a high-pass is obtained from a low-pass, if z^{-1} is replaced by $(-z^{-1})$ and proper potentiation is applied. The same is true for the opposite direction. For the bandwidth extension, it is beneficial to observe that the transitions of the modulation signal additionally are affected by the analog filter preceding analog-to-digital conversion. Thus, in order to characterize the effect of frequency offsets caused by virtual oversampling, the bandwidth extension is advantageously performed in the analog domain after discrete-time to continuous-time conversion. If the bilinear (Tustin) approximation is used for this purpose, distortions added to the frequency response of the low-pass filter generated by the previous steps are really low, because its bandwidth is much smaller than the Nyquist frequency. In the analog domain, it is then easy to extend the bandwidth of the simple low-pass filter by the factor $nVOS \cdot nSpC/k$ postulated above.

This procedure will be exemplified by the calculation of the equivalent low-pass filters that account for the effect of frequency offsets in conjunction with the comb filters used in Fig. 1. The topology of one of these filters is shown in Fig. 3. Its transfer function in z-domain is given by

$$H(z) = \frac{1 - z^{-k}}{1 + g \cdot z^{-k}}. \tag{4}$$

According to the steps detailed above, the comb filter is first turned into a high-pass filter by replacing k delays by one delay:

$$H_{hp}(z) = \frac{1 - z^{-1}}{1 + g \cdot z^{-1}}. \tag{5}$$

The high-pass filter of equation (5) is then converted into a low-pass filter by replacing z^{-1} by $(-z^{-1})$:

$$H_{lp}(z) = \frac{1 + z^{-1}}{1 - g \cdot z^{-1}}. \tag{6}$$

Using the bilinear transformation, this low-pass filter is converted to

$$H_{lp}(s) = \frac{2}{1-g} \cdot \frac{1}{1 + \frac{s \cdot T_S}{2} \frac{(1+g)}{(1-g)}}, \tag{7}$$

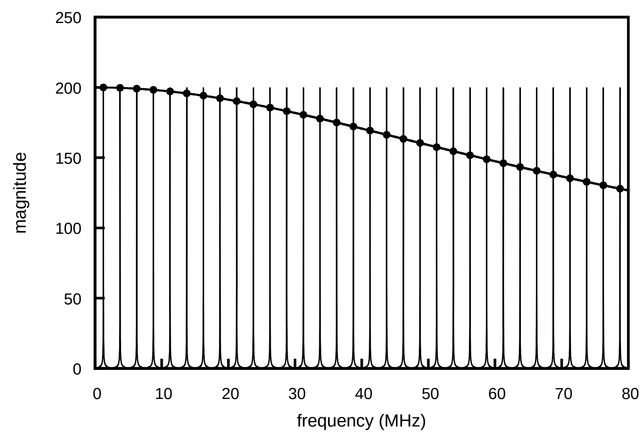

Fig. 4. Magnitude of the comb filter's frequency response (thin solid line), evaluated at the modulation signal's harmonics (dots), and overlaid magnitude of the equivalent low-pass filter's frequency response (thick solid line).

i.e. a filter in continuous time domain, where $T_S = 1/f_S$. Finally, the bandwidth of this filter has to be extended by the factor $nVOS \cdot nSpC/k = 2 \cdot nVOS$ as in

$$H_{equiv}(s) = \frac{2}{1-g} \cdot \frac{1}{1 + \frac{s}{4 \cdot nVOS \cdot f_S} \cdot \frac{(1+g)}{(1-g)}}, \tag{8}$$

to obtain an equivalent low-pass filter that models the effect of frequency offsets in combination with the comb filter frequency response on the harmonics of the modulation signal. This result is visualized in Fig. 4. In this figure, the thin solid line is the magnitude of the comb filter frequency response. This response is sampled at the frequencies of the modulation signal's harmonics as indicated by the dots. If we overlay the discrete time frequency axis with the continuous time frequency axis, we can display the magnitude of the equivalent low-pass filter transfer function (8) as well. It is indicated by the thick solid line in Fig. 4. As shown, the equivalent low-pass filter transfer function of equation (8) approximates the samples of the comb filter transfer function very well.

IV. Simulated Signal Transitions

To demonstrate that the effect of frequency offsets caused by virtual oversampling in combination with the comb filters' frequency responses on the transitions of the modulation signal is accurately represented by effective low-pass filters according to equation (8), a Simulink simulation has been performed. A simple setup as depicted in Fig. 5 has been employed for this purpose. Basically it consists of predefined Simulink blocks that are used to implement two signal channels. The signal channel at the top examines the impact of comb filters on the bipolar clock signal used for modulation in Fig. 1. This signal is generated in continuous time domain by a block called 'Pulse Generator' with its output amplitude set to two, its period length set to l_P and its duty cycle set to 50%. This pulse train is transformed to the bipolar clock signal by a 'Bias' block that subtracts a value of one. Then, in order to obtain comparable signal transitions as in the system of Fig. 1,

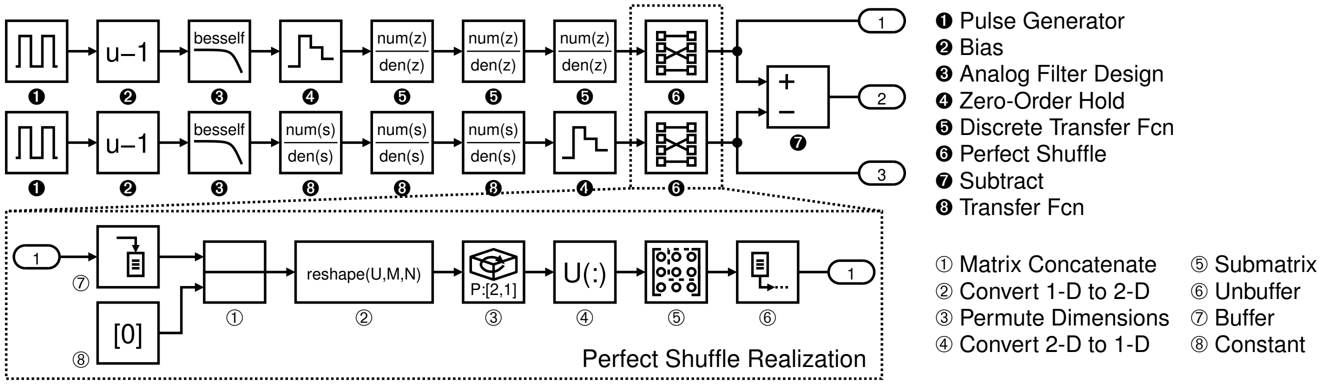

Fig. 5. Simulink block diagram used for comparison between signal transitions influenced by comb filters to those affected by effective low-pass filters.

an 'Analog Filter Design' block implementing a Bessel filter of fifth order with a bandwidth of $1.6216 \cdot 2 \cdot \pi \cdot 25\,\mathrm{MHz}$ is applied. The factor of 1.6216 in the settings is required to re-normalize the filter to the $3\,dB$ bandwidth. The continuous-time to discrete-time conversion is enabled by a 'Zero-Order Hold' block with a sampling interval of $1/f_S$. Three cascaded comb filters are then realized by 'Discrete Transfer Fcn' blocks that are configured according to (4). Finally, a Perfect Shuffle permutation is executed on a frame of samples by the only custom made block to increase the time resolution by the factor of $nVOS$. In this block, as shown in the bottom part of Fig. 5, a 'Buffer' is used to generate a frame of length $nSpC \cdot nVOS - 1$ from the stream of samples. Then a 'Matrix Concatenate' block appends an additional zero to render the frame length a power of two. This allows the 'Convert 1-D to 2-D' block to compose a matrix of $nSpC \times nVOS$ samples from the frame. The matrix is transposed by the 'Permute Dimensions' block and then serialized again by the 'Convert 2-D to 1-D' block, which transfers the samples from the matrix to a frame columnwise. The 'Submatrix' block truncates the additional zero added by the 'Matrix Concatenate' block from the end of the frame that is finally turned into a stream of samples by the 'Unbuffer' block. The signal channel in the center of Fig. 5 works similar to the signal channel at the top, except that it examines the impact of the effective low-pass filters on the signal transitions of the modulation signal. In order to do so, it uses 'Transfer Fcn' blocks configured according to equation (8) prior to continuous-time to discrete-time conversion instead of the 'Discrete Transfer Fcn' blocks described above. In addition to the outputs of both channels, also their difference is observed to allow for a more precise comparison. The results of this simulation are shown in Fig. 6. The diagram at the top displays the output of the channel that contains the actual comb filters. Their settling behavior can be identified roughly in the time from $0.1\,ms$ to $0.4\,ms$. The diagram in the center of Fig. 6 shows the output of the channel that uses effective low-pass filters instead. They have a much shorter settling time and thus provide full output signal levels early during the simulation run. This is why the difference between both outputs in the bottom diagram of Fig. 6 exhibits

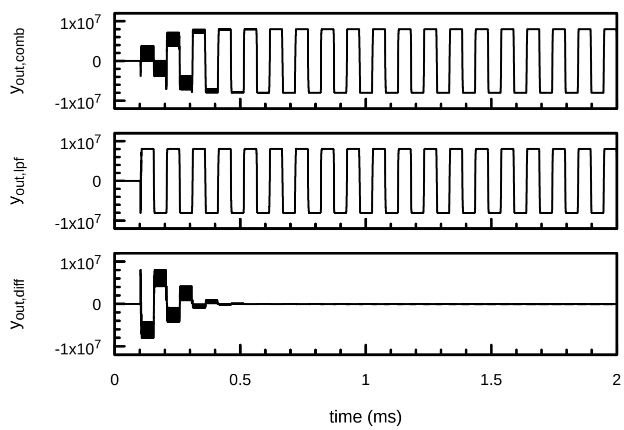

Fig. 6. Simulation results: Output of the channel with real comb filters (top), output of the channel with effective low-pass filters (center) as well as their difference (bottom).

larger values in the range from roughly $0.1\,ms$ to $0.4\,ms$ but converges to negligible values afterwards.

V. Conclusion

In this paper it has been shown, that the harmonics of a bipolar modulation signal in a specific localization system sample the frequency responses of comb filters at distinct frequencies – ideally at transmission band centers. However, if virtual oversampling is applied to increase the effective sampling rate, offsets occur, that shift the harmonics progressively out of their ideal frequency locations. This effect can be modeled by an effective low-pass filter transfer function in continuous-time domain. By comparison it has been shown that the effective low-pass filter transfer functions can account for the effect of frequency offsets on the transitions of the modulation signal. This observation allows for additional signal processing algorithms that require knowledge of the shape of signal transitions.

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